# The Correlation Of A Random Variate Plus A Constant With Another Random Variate

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Question: Is the correlation between the random variates x and y and the correlation between the random variate x ***plus a constant*** and the random variate y the same?

## The Mean and Variance of the Random Variate X

The mean of the random variate x is...

$$\mu_x = \mathbb{E}\Big[x\Big] \tag{1}$$

The variance of the random variate x is...

$$\sigma_x^2 = \mathbb{E}\Big[x^2\Big] - \Big[\mathbb{E}\Big[x\Big]\Big]^2 \tag{2}$$

## The Mean and Variance of the Random Variate Y

The mean of the random variate y is...

$$\mu_y = \mathbb{E}\Big[y\Big] \tag{3}$$

The variance of the random variate y is...

$$\sigma_y^2 = \mathbb{E}\Big[y^2\Big] - \Big[\mathbb{E}\Big[y\Big]\Big]^2 \tag{4}$$

## The Correlation of the Random Variates X and Y

The covariance between the random variates x and y is...

$$cov_{xy} = \mathbb{E}\Big[(x - \mu_x)(y - \mu_y)\Big] \tag{5}$$

The correlation of the random variates x and y is...

$$\rho_{xy} = \frac{cov(x, y)}{\sigma_x \sigma_y} \tag{6}$$

## The Mean and Variance of the Random Variate X Plus a Constant

We will define the variable z to be the random variate x plus a constant such that...

$$z = x + c \tag{7}$$

The mean of the random variate z is...

$$\begin{aligned}\mu_z &= \mathbb{E}\Big[z\Big] \\ &= \mathbb{E}\Big[x + c\Big] \\ &= \mathbb{E}\Big[x\Big] + c \\ &= \mu_x + c \qquad (8)\end{aligned}$$

The variance of the random variate z is...

$$\begin{aligned}\sigma_z^2 &= \mathbb{E}\Big[z^2\Big] - \Big[\mathbb{E}\Big[z\Big]\Big]^2 \\ &= \mathbb{E}\Big[(x+c)^2\Big] - \Big[\mathbb{E}\Big[x + c\Big]\Big]^2 \\ &= \mathbb{E}\Big[x^2 + 2cx + c^2\Big] - \Big[\mu_x + c\Big]^2 \\ &= \mathbb{E}\Big[x^2\Big] + \mathbb{E}\Big[2cx\Big] + \mathbb{E}\Big[c^2\Big] - \Big[\mu_x^2 + 2c\mu_x + c^2\Big] \\ &= \mathbb{E}\Big[x^2\Big] + 2c\,\mathbb{E}\Big[x\Big] + c^2 - \mu_x^2 - 2c\mu_x - c^2 \\ &= \mathbb{E}\Big[x^2\Big] + 2c\mu_x + c^2 - \mu_x^2 - 2c\mu_x - c^2 \\ &= \mathbb{E}\Big[x^2\Big] - \mu_x^2 \\ &= \mathbb{E}\Big[x^2\Big] - \Big[\mathbb{E}\Big[x\Big]\Big]^2 \\ &= \sigma_x^2 \qquad (9)\end{aligned}$$

The covariance of the random variable z and the random variate y is...

$$cov_{zy} = \mathbb{E}\Big[(z - \mu_z)(y - \mu_y)\Big] \qquad (10)$$

We can replace the variable z and $\mu_z$ in equation (10) above with equations (7) and (8) such that equation (10) becomes...

$$\begin{aligned}cov_{zy} &= \mathbb{E}\Big[(x + c - [\mu_x + c])(y - \mu_y)\Big] \\ &= \mathbb{E}\Big[(x - \mu_x)(y - \mu_y)\Big] \\ &= cov_{xy} \qquad (11)\end{aligned}$$

## Are the Correlations the Same?

We will start with the equation (6), which is the correlation between x and y...

$$\rho_{xy} = \frac{cov(x, y)}{\sigma_x \sigma_y} \qquad (12)$$

After noting that the covariance of x and y equals the covariance of z and y we can replace the $cov(x, y)$ in equation (12) above with equation (11) such that equation (12) becomes...

$$\rho_{xy} = \frac{cov(z, y)}{\sigma_x \sigma_y} \qquad (13)$$

After noting that $\sigma_x^2$ equals $\sigma_z^2$ per equation (9) above we can replace the $\sigma_x$ in equation (13) with $\sigma_z$ such that equation (13) becomes...

$$\rho_{xy} = \frac{cov(z, y)}{\sigma_z \sigma_y} \tag{14}$$

After noting that the right side of equation (14) above is the correlation of the random variates z, which is x plus a constant, and y equation (14) becomes...

$$\rho_{xy} = \rho_{zy} \tag{15}$$

Conclusion: We have proved that the correlation between the random variates x and y and the correlation between the random variate x plus a constant and the random variate y is the same.